Galactic Rotation Curves and Linear Potential Laws

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Abstract

We study the possibility that galactic rotation curves can be explained by a gravitational potential that contains a linear term as well as a Newtonian one. This hypothesis, suggested by conformal gravity, does allow good fits to the galactic rotation curves of the galaxies we study, which have a wide range of luminosities. However, the universality one might have expected of the parameter describing the strength of the linear potential does not emerge. Instead, a different regularity is seen.

1 Introduction

The flat rotation curve seen in many galaxies cannot be explained by the visible matter and a Newtonian potential. Often this is taken as evidence for non-visible, i.e., "dark," matter in the galaxies. There are many candidates for what the dark matter could be (see for example Griest (1995)). Indeed, one of the remarkable sucesses in the past year has been the observation of one category of these candidates, the MACHOs or massive compact halo objects by the EROS (Aubourg *et al.* 1993, 1995; Beaulieu *et al.* 1995), OGLE (Udalski *et al.* 1993, 1994a,b,c), and MACHO (Alcock *et al.* 1993, 1995a,b,c; Bennett *et al.* 1994) collaborations. Unfortunately, these were discovered to be insufficiently numerous to explain the flat rotation curves.

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Whatever one's hopes or prejudices, it is important to consider alternatives. In the present case, one alternative is that dark matter is not the cause of the rotation curve problem, but rather that the gravitational forces are not Newtonian at long distances. For example, Milgrom (1983, 1986, 1988, 1994) has suggested a modified Newtonian dynamics wherein the gravitational acceleration is the usually calculated Newtonian gravitational acceleration if that acceleration is large compared to some critical acceleration, but is the geometrical mean of the critical acceleration and the calculated Newtonian accleration if the Newtonian acceleration is very small. Fits to galactic data, after some early doubts, seem possible using the same critical acceration for each galaxy (Begeman, Broeils, & Sanders 1991). Milgrom's idea gives an acceleration like what one would get from a $1/r$ force, proportional to the square root of the mass, at very long range.

Here we shall examine a different alternative, namely that, in addition to the Newtonian term, the potential also contains a linear term. This has been suggested on the basis of conformal gravity (Mannheim & Kazanas 1989, 1991; Mannheim 1993), but one does not need to subscribe to this viewpoint in order to evaluate the result. For a point mass M, the potential (potential energy per unit test mass) is

$$V = -\frac{GM}{r} + \Gamma M r. \tag{1}$$

The linear term must have a very small coefficient so that it will contribute noticeably only at very long distances. The coefficient of the linear potential, Γ, should, like G, be a universal coefficient if this potential correctly describes nature.

We shall select a group of galaxies for which a rotation curve has been well measured, and for which sufficient other data is available that can calculate a rotation curve based on the directly observed matter. We shall, at least at the outset, allow ourselves to vary only the mass to light ratio of the luminous disk and the value of the linear potential coefficient Γ. The first goal is to see if good fits to the rotation curve can be obtained. If the answer is generally yes, then the further question will be whether good fits can be obtained with the same value of Γ for each galaxy. The answer will be seen to be that good fits can be obtained, but not with a universal value of Γ. However, we will notice that a fairly but not absolutely consistent value of $\Gamma \times M_{galaxy}$ does emerge. The latter is equivalent to saying that when the gravitational forces

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get very small, one does not get the Newtonian gravitational acceleration but rather some small limiting value of acceleration. Finally, there will be some short discussion of what is possible if some other parameters, such as the distance to the galaxy, are allowed to vary.

2   Testing the idea

The galaxies we use are the ten used by Begeman *et al.* (1991). There are optical measurements giving the luminosity, scale length, and distance of each of these galaxies, and also radio measurements of the rotation curve many scale lengths out from the center of the galaxy. The number of galaxies is restricted (Begeman *et al.* 1991) by a requirement that they have reasonable azimuthal symmetry so that the rotation curves accurately trace the overall mass distribution of the galaxies. The galaxies we use, as well as their distances, luminosities, and scale lengths, are listed in Table 1. The galaxies differ by a factor of 1000 in luminosity and a factor 10 in size (scale length). We will not, at least for now, vary the distances and scale lengths found in the literature.

We give a few details of how we proceed.

The visible mass of each galaxy includes luminous matter and Hɪ gas. The luminosity area density of the galactic disk is generally well represented by a falloff that is exponential in distance from the galactic center. The scale lengths R_D are given in Table 1. We take the mass to light ratio for the luminous matter in the disk, M/L, to be constant within a given galaxy although we allow it to vary from galaxy to galaxy. The mass area density of the luminous disk, σ_D, is then

$$\sigma_D = \left(\frac{M_D}{2\pi R_D^2}\right) e^{-r/R_D}. \tag{2}$$

where M_D is the mass of the luminous matter in the disk.

The point mass Newtonian plus linear potential needs to be integrated over the expontential disk mass distribution to get the potential for a galaxy. Mannheim (1993, 1995) has given the result extending earlier work by others for the Newtonian case. The Newtonian force per unit test mass (acceleration) is

$$g_{ND} = \frac{GM_D}{R_D^2}\alpha \left[I_0(\alpha)K_0(\alpha) - I_1(\alpha)K_1(\alpha)\right] \qquad (3)$$

where

$$\alpha \equiv r/2R_0. \qquad (4)$$

and I_ν and K_ν are Bessel functions. The corresponding result for the linear potential is

$$g_{LD} = 2\Gamma M_D \alpha I_1(\alpha)K_1(\alpha). \qquad (5)$$

Two of the galaxies, the two biggest, in our sample have central bulges which are not well included by a single exponential falloff. For these galaxies the luminosity curve is fit with a sum of two exponentials. The bulge and (main) disk components are allowed different M/L ratios, and in Table 1 we list the scale lengths, fitted M/L ratios, and total mass of the two components separately.

In addition, the H$\textsc{i}$ gas in the galaxies is visible through its 21 cm radiation. If the distance to the galaxy is correctly known, the measurements give the mass of the H$\textsc{i}$ directly, and we increase this mass by a factor 4/3 to account for heavier material, mainly primordial He. The gas contribution is dynamically significant only for the three lightest galaxies in our sample, and for each of these the gas mass area density is well fit by a gaussian,

$$\sigma_G = \frac{M_G}{\pi R_G^2}e^{-r^2/R_G^2}, \qquad (6)$$

and the total gas mass M_G and gaussian scale length for the gas R_G are listed in Table 2. We here record the acceleration fields due to the gas and the Newtonian and linear potentials (Mannheim 1995):

$$g_{NG} = \frac{GM_G r}{R_G^3}\sqrt{\pi}e^{-\beta}\left(I_0(\beta) - I_1(\beta)\right) \qquad (7)$$

where

$$\beta \equiv r^2/2R_G^2 \qquad (8)$$

and

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$$g_{LG} = \frac{\Gamma M_G r}{2 R_G} \sqrt{\pi} e^{-\beta} \left(I_0(\beta) + I_1(\beta) \right). \tag{9}$$

Thus in a case where we include gas and a luminous disk described by a single exponential we have

$$g(r) = \frac{v^2(r)}{r} = g_{ND} + g_{NG} + g_{LD} + g_{LG}, \tag{10}$$

where $v(r)$ is the revolution velocity of the galaxy.

We then fit to the galactic rotation curves, given the distribution and luminosity of the luminous matter and the distribution and actual mass of the gas. To repeat, the unknown quantities for each galaxy are M/L for the luminous matter and Γ. The rise of the rotation curve to its peak is mainly determined by the Newtonian part of the gravitational force (the contribution from the linear potential is numerically small in this region), and this rise determines M/L. Conversely, the contribution of the Newtonian potential to the far out part of the rotation curve is small, and this part of the curve basically determines Γ for each galaxy.

Our fits to the rotation curves are shown in Fig. 1. One sees that the rotation curve fits, based on two parameters (three for NGC 2841 and 7331), are tolerably good. The curves in Fig. 1 also show a clear feature of a linear plus Newtonian potential in that the observed near-flatness depends upon interplay between the two contributions, and if the rotation curve were measured farther out the curve would rise.

The values of M/L and Γ we get for each galaxy are shown in Table 1. Although we have good fits, the price is that the largest and smallest values of Γ in the Table differ by two orders of magnitude. We can get an idea of the play in Γ by asking what values we get if we set M/L to zero (which gives maximum possible Γ at the expense of a poor small radius fit), or set M/L to twice its best value (which also gives a poor small radius fit). One gets changes of about $\pm 5\%$ in Γ for the gas dominated galaxy DDO 154 to typically $\pm 30\%$ for a galaxy where the known gas plays little dynamical rôle. Thus the fits cannot be modified to get the same Γ for each galaxy and we conclude that the values of Γ are not universal.

2.1 Allowing the distance to vary

Distances to galaxies are of course not perfectly measured. Indeed, for DDO 154 there is discussion (Carignan & Beaulieu 1989) of whether it is part of the Canes Venatici I cluster at 4 Mpc or really part of the Coma I cluster beyond it at 10 Mpc. So we may consider how changes in the measured distance will affect the values of Γ and $M_{gal}\Gamma$. For galaxies dominated by luminous matter, when the distance scales like $d \to \eta d$, then $r \to \eta r$ and $L \to \eta^2 L$. Then choosing $(M/L) \to \eta^{-1}(M/L)$ leads to unchanged rotation curves provided

$$\Gamma_{new} = \eta^{-2}\Gamma_{old} = \left(\frac{d_{old}}{d_{new}}\right)^2 \Gamma_{old}. \tag{11}$$

For gas dominated galaxies, we have directly $M \to \eta^2 M$ and examining the far out part of the rotation curve then leads to

$$\Gamma_{new} = \left(\frac{d_{old}}{d_{new}}\right)^3 \Gamma_{old}. \tag{12}$$

For either case,

$$(M_{gal}\Gamma)_{new} = \frac{d_{old}}{d_{new}}(M_{gal}\Gamma)_{old}. \tag{13}$$

To reconcile or make the same all the values of Γ would involve moving galactic distances so that the smaller galaxies were systematically moved out by a factor of 5 to 10 compared to the larger. We will not entertain this idea. Reconciling $M_{gal}\Gamma$ is less motivated theoretically, but it is relatively easy to do. Choosing to set $M_{gal}\Gamma = 2.42$ (the geometric mean of the relevant numbers in Table 1), a two parameter fit varying d and M/L produces rotation curves like the ones we have already shown, with the scaling of $M_{gal}\Gamma$ working as suggested above even for the cases where luminous matter and gas are both important. Reconciling $M_{gal}\Gamma$ in this way requires, in the extreme cases, having DDO 154 at 2.4 Mpc instead of 4 Mpc and NGC 2841 at 22.5 Mpc instead of 9.46 Mpc. In fact, for NGC 2841 the distance is already in dispute (Sanders & Begeman 1994) since the distance derived from the Tully-Fisher relation is about twice 9.46 Mpc obtained from Hubble's law and used here. Sanders & Begeman (1994) suggest 2841's recession speed is greatly affected

by proximity to the Virgo Cluster, and that the larger distance is more likely correct.

3 Conclusion

We have attempted to confirm or disconfirm the idea that the far out part of the galactic rotation curves, usually taken as evidence of dark matter, may be well described by a linear add-on to the Newtonian potential. The original suggestion was that the coefficient of the linear potential be a universal constant Γ times the mass of the source, just as the coefficient in the Newtonian potential is a universal constant G times the mass of the source. This is what one would expect if gravity theory, even with modifications, were a metric theory driven by the energy-momentum tensor of the source.

The original suggestion works poorly. Decent fits can be gotten for the rotation curves of a selection of smooth and azimuthally symmetric galaxies, but the fits require very different values of Γ for different size galaxies.

The galactic data, despite lacking *a priori* theoretical motivation, do seem to give $M_{gal} \times \Gamma$ nearly the same for each galaxy. In other words, the centripetal acceleration of matter near the galactic edges approaches a limiting value which is about the same for any galaxy. Numerically, the value $g_0 = M_{gal}\Gamma$ is about $2\frac{1}{2} \times 10^{-11} m/s^2$. Thus there is some systematic and reproducible feature of the mysterious galactic rotation curves. Such things have also been noted in the context of the dark matter explanation (Bahcall and Casertano 1985; van Albada & Sancisi 1986), and may be a clue to an underlying understanding of galactic structure or binding.

Acknowledgment

Galaxy	d Mpc	L $10^9 L_\odot$	R_D kpc	M/L $M_\odot/L_\odot$	Γ $10^{-52} N/kg^2$	M_{gal} $10^9 M_\odot$	ΓM_{gal} $10^{-11} m/s^2$
DDO 154	4.00	0.05	0.50	1.2	164	0.42	1.37
DDO 170	12.01	0.16	1.28	4.5	46	1.6	1.46
NGC 1560	3.00	0.35	1.30	2.3	60	1.9	2.27
UGC 2259	9.80	1.02	1.33	4.1	29	4.2	2.38
NGC 6503	5.94	4.80	1.73	3.0	5.8	14	1.67
NGC 2403	3.25	7.90	2.05	2.3	6.9	18	2.49
NGC 3198	9.36	9.00	2.63	3.8	3.1	34	2.14
NGC 2903	6.40	15.30	2.02	3.6	3.0	55	3.31
NGC 2841	9.46	20.50	0.50	3.5	1.9	150	5.75
		(5.9+14.6)	2.38	9.0			
NGC 7331	14.90	54.00	1.20	0.75	1.3	140	3.71
		(31.5+22.5)	4.48	5.2			

Table 1: The galaxies. The two brightest galaxies have two component expontentials describing their luminosity profiles. Both scale lengths are given, and the luminosities of the inner ("bulge") and outer parts of the disk are given, in that order, parenthetically in the luminosity column.

Galaxy	$M_{\rm HI}(M_\odot)$	$M_G(M_\odot)$	R_G
DDO 154	2.7×10^8	3.6×10^8	$3.3' = 3.8$kpc
DDO 170	6.6×10^8	8.8×10^8	$95'' = 6.7$kpc
NGC 1560	8.2×10^8	10.9×10^8	$5.6' = 4.85$kpc

Table 2: Parameters for gas in three galaxies.

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Figure 1: *Caption:* Rotation curves. In all cases, v is in km/sec and r is in kpc. The heavy line is the full result. The dotted line would be the result from the linear potential alone, the solid line would be the result from the Newtonian potential alone. In the cases where there is a two component fit to the mass distribution, we have indicated the separate contributions to the Newtonian result with dashed lines. For the lighter galaxies, the two components are luminous matter and gas, with the gas contribution peaking farther out; for the heavier galaxies, both components represent luminous matter.